Good afternoon.

I’m Avi, I am from the robotics lab at Ben Gurion University

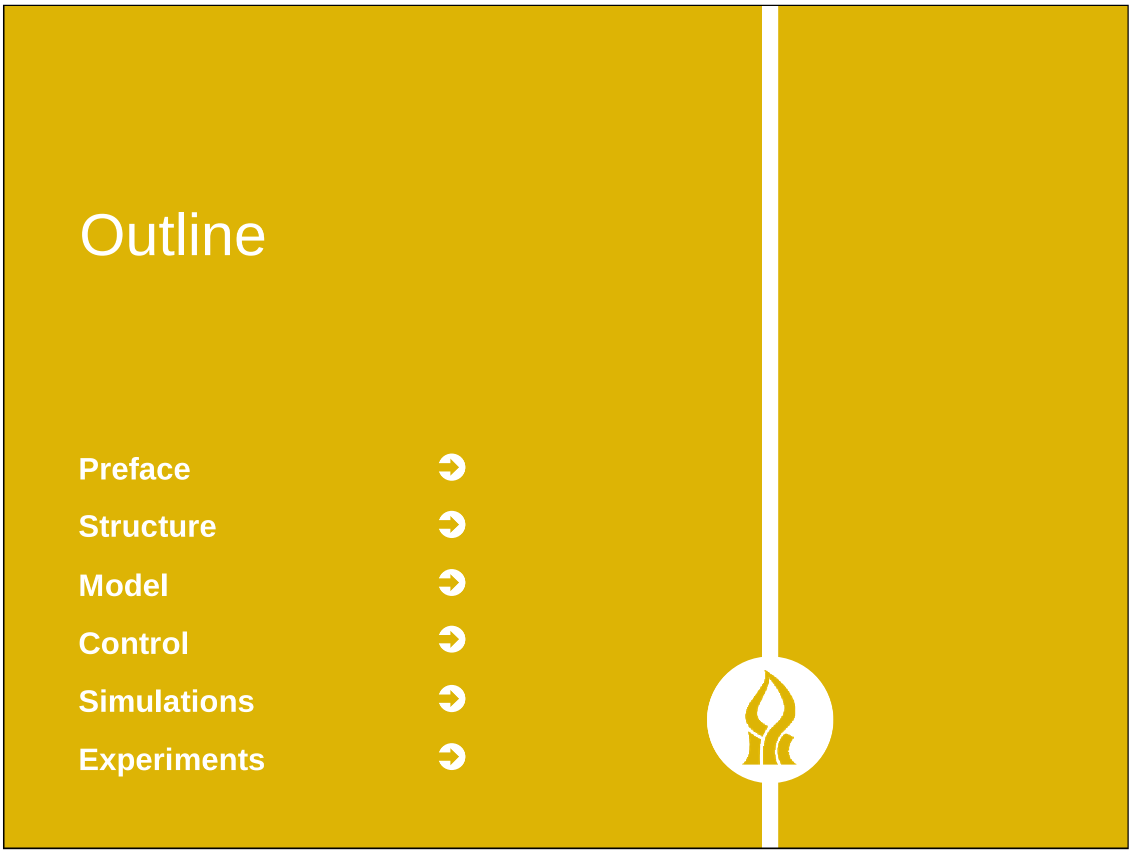
I am going to present to you today a research work I have done under the supervision of Dr. Amir Shapiro

The title of my talk is “How to get FAR”

- **How to get FAR**
  this not a new age topic it's an aircraft!
- **Fully Actuated Rotorcraft (FAR)**
- **Unique**
  No other aircraft with that ability

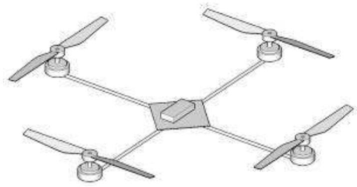
FAR stands for Fully Actuated Rotorcraft

FAR is unique: there is no other aircraft or helicopter that can change its position without changing its orientation and vice versa



Today I am going to talk about the
Structure of the FAR
Its analytical model
How we developed its control system
How we used simulations
and finally I will show some experiments results

Shortcomings of quadrotors



- **How quads work?**
- **Coupling between DOFs**
  no big deal, you can do amazing things with quads
  see ETH Zurich, Vijay Kumar Penn.
- **There are strict bounds to the torque**
  exceeding the bounds will cause a loss of a control DOF
  Effects the quads proportions and sets a design limit
- **Platform span to rotor length**
  long quads are not well behaved because of low yaw torque
- **Shrouded rotors make things worse**

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Due to the lack of time I am not going to elaborate about quadrotores:

But it seems that you can find quadrotors today everywhere and people are doing really amazing things with them.

Tough none of them can do what I am going to show you today.

Just as a reminder;

Quadrotors have 4 actuators and like most aircrafts, 6 degrees of freedom.

So they are under actuated.

They can not change position without changing orientation.

A less known fact about quadrotors is that the reactive torque used to control the yaw of the platform is very small.

It is about a 100 times smaller than the thrust generated by the rotors.

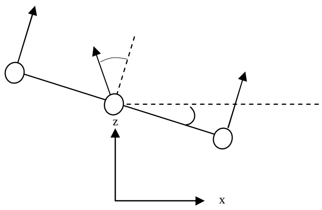
Shrouding the rotors will decrease the reactive torque even further.

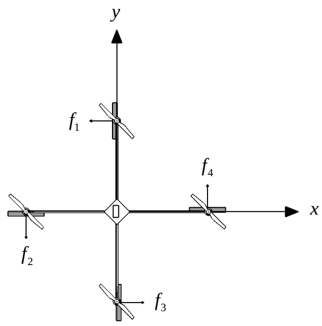
Because of that it is difficult to control a long quad fitted with small rotors.

Actually if just try to make a long quadrotor to get the flow away from the interference of the hub; you will immediately encounter difficulties because the inertia changes to the fifth 5th power of length.

How to get FAR?

- Thrust vectoring as an idealized concept
- Add a single DOF for each of the 4 propulsion units (rotor)
- Arrange a control scheme to create 6 independent DOFs
- Selection of thrust vectoring DOF





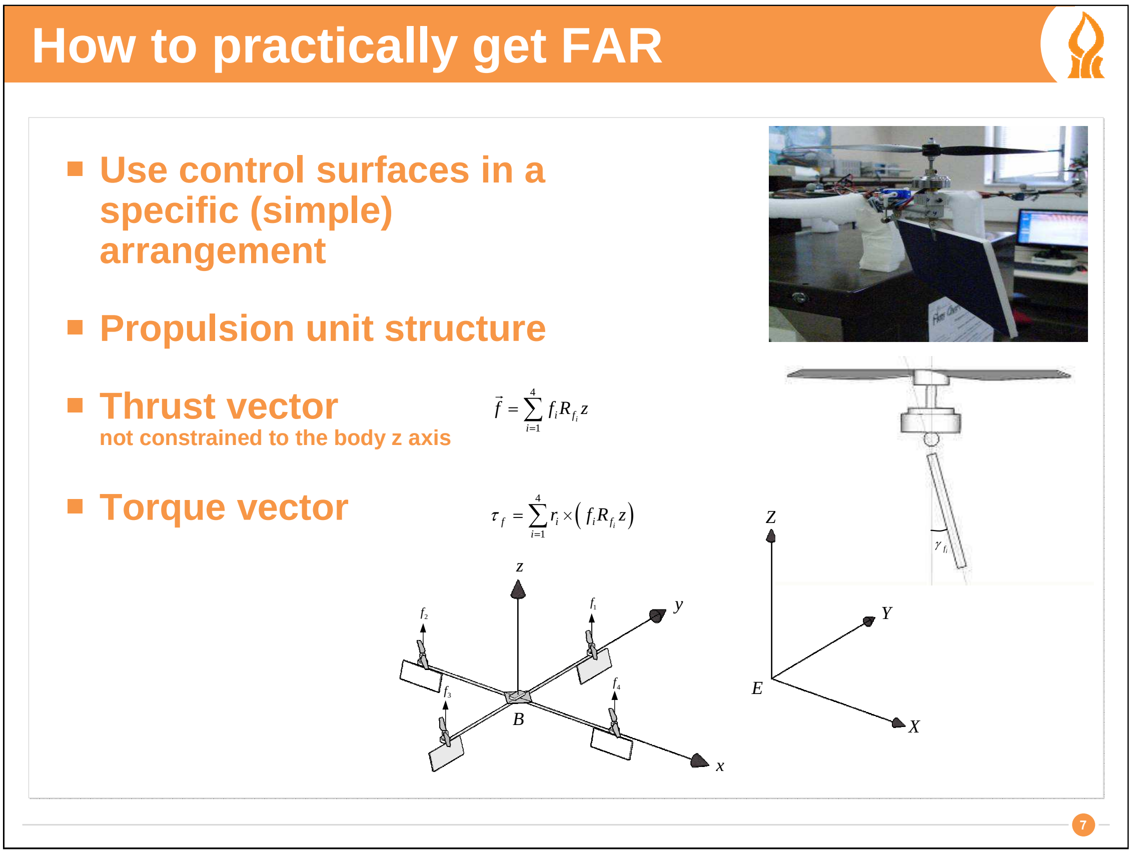
To create the FAR we used thrust vectoring.

We added a thrust deflection actuator to each of the rotors and arranged the 8 actuation inputs to get 6 independent control inputs.

We chose to deflect the thrust so it will have a component perpendicular to the rods the motors are mounted on as presented in the drawings at bottom of the screen.

Structure

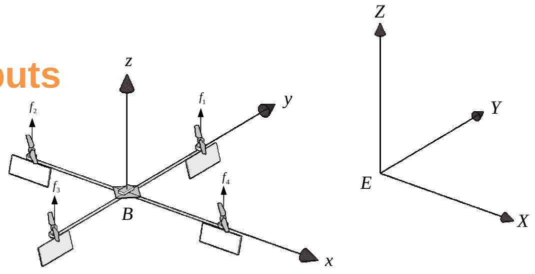
The FAR structure!



To deflect the thrust we added a control surface underneath each of the rotors.
You can see how it was done in the experimental system in the picture
and may be a bit more clearly in the drawing below it.
The outcome of this is that the thrust is no longer bound to the Z axis of the platform.

Arrangement of control inputs

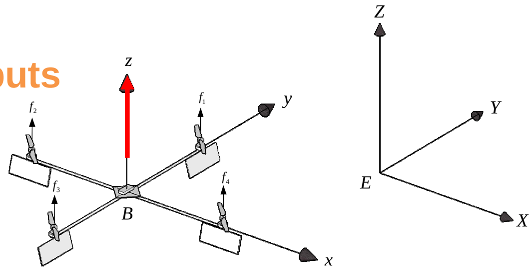
- 8 actuators
- Arranged as 6 control inputs
  - Thrust
  - 2 X torque
  - 3 X thrust deflection angles



Now Let’s see how we arranged the 8 actuators into 6 control inputs:

Arrangement of control inputs

- 8 actuators
- Arranged as 6 control inputs
  - Thrust
  - 2 X torque
  - 3 X thrust deflection angles

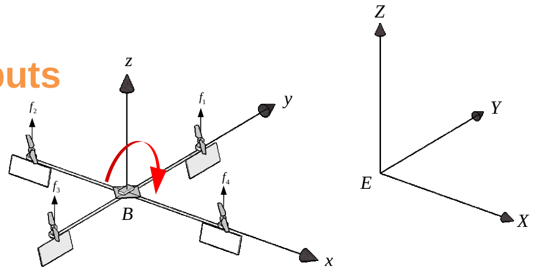


$$u_1 = f_1 + f_3 = f_2 + f_4$$

1. Thrust – similar to a regular quad except the thrust of the couples F1-F3 and F2-F4 are constrained to be equal.

Arrangement of control inputs

- 8 actuators
- Arranged as 6 control inputs
  - Thrust
  - 2 X torque
  - 3 X thrust deflection angles

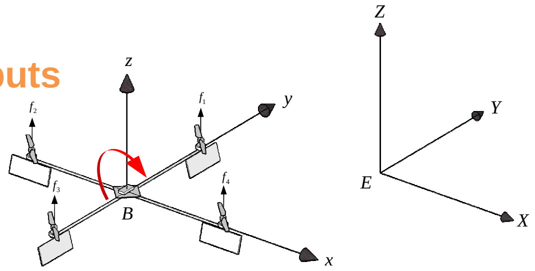


$$u_1 = f_1 + f_3 = f_2 + f_4$$
$$u_2 = f_2 - f_4$$

2. Roll and Pitch torques are achieved in the same way they are achieved in quads by creating a difference in the thrust along the main axis of the platform.

Arrangement of control inputs

- 8 actuators
- Arranged as 6 control inputs
  - Thrust
  - 2 X torque
  - 3 X thrust deflection angles



$$u_1 = f_1 + f_3 = f_2 + f_4$$
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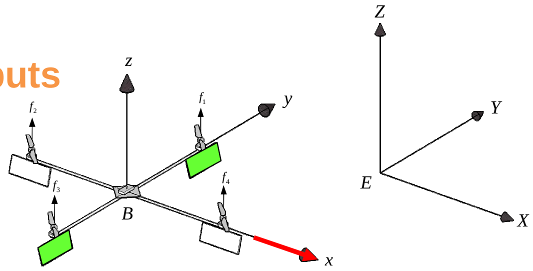
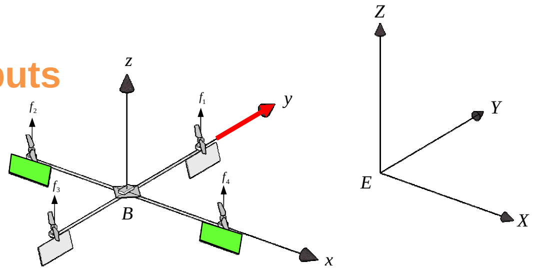


$$u_1 = f_1 + f_3 = f_2 + f_4$$
$$u_2 = f_2 - f_4$$
$$u_3 = f_1 - f_3$$
$$u_4 = \gamma_1; \quad \gamma_1 = \frac{\gamma_{f_1} + \gamma_{f_3}}{2}$$

3. Force along the platform X axis is produced by deflecting the thrust of F1 and F3.

Arrangement of control inputs

- 8 actuators
- Arranged as 6 control inputs
  - Thrust
  - 2 X torque
  - 3 X thrust deflection angles

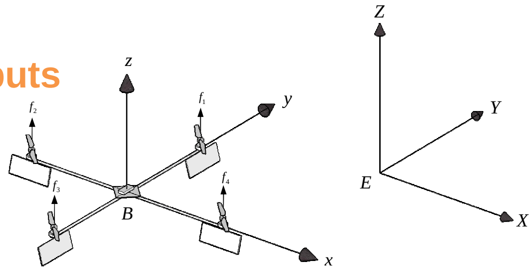


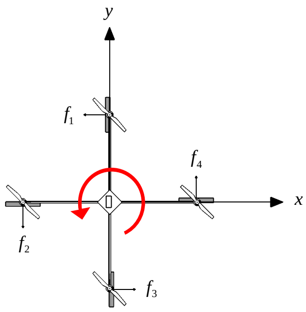
$$u_1 = f_1 + f_3 = f_2 + f_4$$
$$u_2 = f_2 - f_4$$
$$u_3 = f_1 - f_3$$
$$u_4 = \gamma_1; \quad \gamma_1 = \frac{\gamma_{f_1} + \gamma_{f_3}}{2}$$
$$u_5 = \gamma_2; \quad \gamma_2 = \frac{\gamma_{f_2} + \gamma_{f_4}}{2}$$

4. Similarly force along the platform Y axis is produced by deflecting the thrust of F2 and F4.

Arrangement of control inputs

- 8 actuators
- Arranged as 6 control inputs
  - Thrust
  - 2 X torque
  - 3 X thrust deflection angles





$$u_1 = f_1 + f_3 = f_2 + f_4$$
$$u_2 = f_2 - f_4$$
$$u_3 = f_1 - f_3$$
$$u_4 = \gamma_1;\quad \gamma_1 = \frac{\gamma_{f_1} + \gamma_{f_3}}{2}$$
$$u_5 = \gamma_2;\quad \gamma_2 = \frac{\gamma_{f_2} + \gamma_{f_4}}{2}$$
$$u_6 = \gamma_3;\quad \gamma_3 = \frac{\gamma_{f_1} - \gamma_{f_3} + \gamma_{f_2} - \gamma_{f_4}}{4}$$

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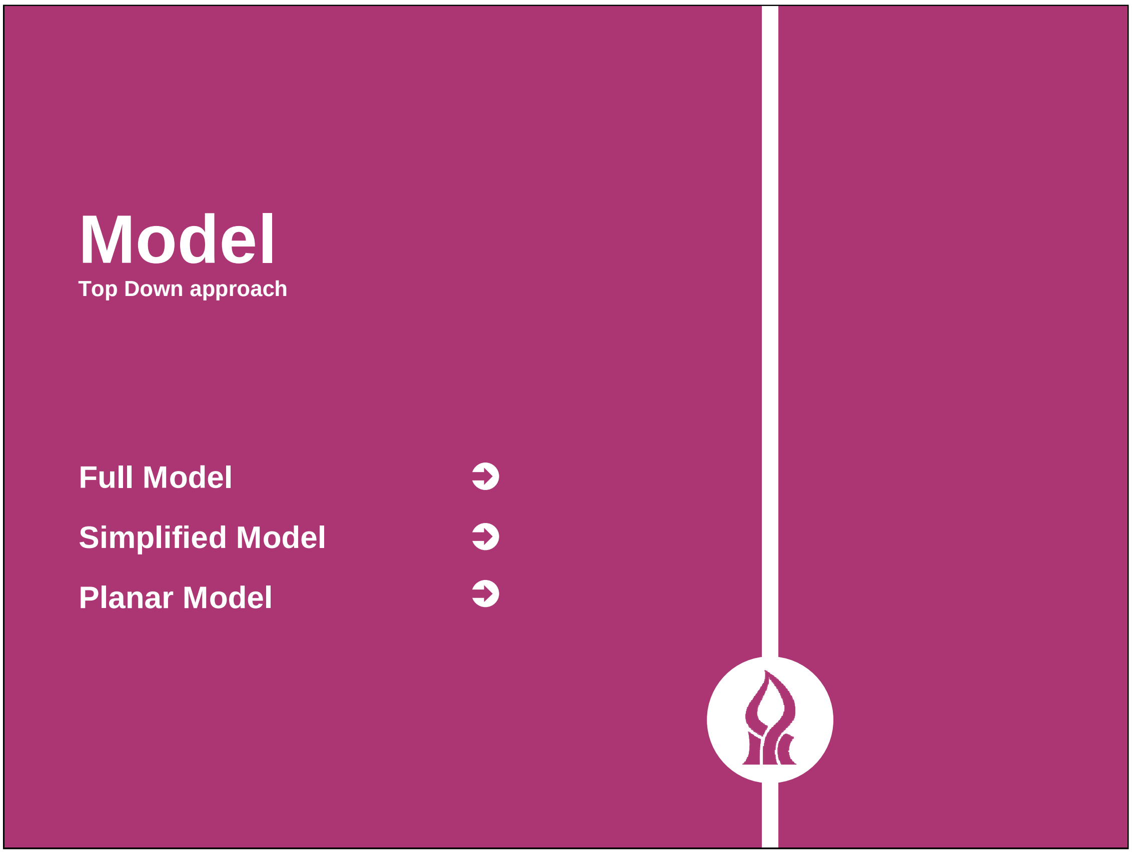
5. To create a yaw torque the thrust of all the rotors is deflected in clockwise or counter Clockwise way around the Z axis

This is done in the way presented in the drawing below

Let’s see how we model the FAR



The FAR model

When we analyzed the FAR model towards the design of the control system we used a top down approach.

Let me show you what I mean

Top-Down Approach

The Full Model

- approximations and assumptions

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We started by building the Full model of the aircraft.

The full model includes:

Top-Down Approach

$$\dot{v} = -g\hat{z} + \frac{1}{m}R\vec{f} + f_d(v)$$

The Full Model

- approximations and assumptions

1. The linear dynamics

Top-Down Approach

$$\dot{v} = -g\hat{z} + \frac{1}{m}R\vec{f} + f_d(v)$$
$$I_f\dot{\Omega} = -\Omega \times I_f\Omega - G_a + \tau_a + \tau_d(\Omega)$$
$$\tau_a = R(\tau_f + \tau_Q)$$

The Full Model

- approximations and assumptions

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2. The angular dynamics

Top-Down Approach

$$\dot{v} = -g\hat{z} + \frac{1}{m}R\vec{f} + f_d(v)$$
$$I_f\dot{\Omega} = -\Omega \times I_f\Omega - G_a + \tau_a + \tau_d(\Omega)$$
$$\tau_a = R(\tau_f + \tau_Q)$$
$$I_r\dot{\omega}_i = \frac{K_t}{R_a}v_{ai} - \left(b_m + \frac{K_tK_e}{R_a}\right)\omega_i - k\omega_i^2$$

The Full Model

- approximations and assumptions

3. The dynamics of the propulsion system

Top-Down Approach

$$\dot{v} = -g\hat{z} + \frac{1}{m}R\vec{f} + f_d(v)$$
$$I_f\dot{\Omega} = -\Omega \times I_f\Omega - G_a + \tau_a + \tau_d(\Omega)$$
$$\tau_a = R(\tau_f + \tau_Q)$$
$$I_r\dot{\omega}_i = \frac{K_t}{R_a}v_{ai} - \left(b_m + \frac{K_tK_e}{R_a}\right)\omega_i - k\omega_i^2$$
$$I_\gamma\ddot{\gamma}_i = \tau_{\gamma_i}$$

The Full Model

- approximations and assumptions

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4. And the dynamics of the thrust deflection actuators

Than we used assumptions and approximations to construct a simplified model.

Top-Down Approach

$$\dot{v} = -g\hat{z} + \frac{1}{m} R\vec{f} + f_d(v)$$

$$I_f \dot{\Omega} = -\Omega \times I_f \Omega - G_a + \tau_a + \tau_d(\Omega)$$

$$\tau_a = R(\tau_f + \tau_Q)$$

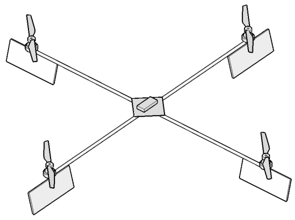
$$I_r \dot{\omega}_i = \frac{K_t}{R_a} v_{ai} - \left(b_m + \frac{K_t K_e}{R_a} \right) \omega_i - k\omega_i^2$$

$$I_\gamma \ddot{\gamma}_i = \tau_{\gamma_i}$$

$$\dot{v} = -g\hat{z} + \frac{1}{m} \vec{f}$$

$$I_f \dot{\Omega} = \tau_f$$

The Full Model → **Simplified Model**



- approximations and assumptions

We assumed:

A symmetrical structure.

That the linear and angular velocities are slow

That The FAR in near stable hover

and that the actuators dynamics is much faster than the dynamics of the platform

And we got the simplified model.

Top-Down Approach

$$\dot{v} = -g\hat{z} + \frac{1}{m}R\vec{f} + f_d(v)$$
$$I_f\dot{\Omega} = -\Omega \times I_f\Omega - G_a + \tau_a + \tau_d(\Omega)$$
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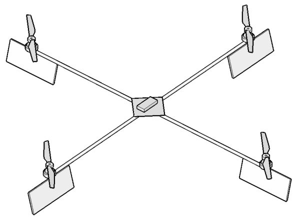
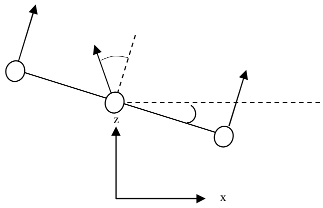
$$I_\gamma\ddot{\gamma}_i = \tau_{\gamma_i}$$

$$\dot{v} = -g\hat{z} + \frac{1}{m}\vec{f}$$
$$I_f\dot{\Omega} = \tau_f$$

$$M\ddot{x} = (f_1 + f_3)S_{\theta+\gamma_1} + (f_2 + f_4)S_\theta$$
$$M\ddot{z} = (f_1 + f_3)C_{\theta+\gamma_1} + (f_2 + f_4)C_\theta - Mg$$
$$I_x\ddot{\theta} = a(f_2 - f_4)$$

The Full Model → **Simplified Model** → **Simplified planar model**





- approximations and assumptions

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To gain further intuitive understanding of the model we constrained it to a plane and formed the Simplified planar model.

Control

Bottom Up approach

For designing the FAR control system we used a Bottom Up approach

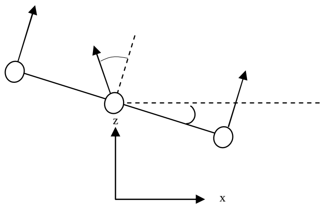
Bottom-up approach – Controllers

$$u_1 = \frac{M}{2C_{\theta+\frac{\gamma_1}{2}}C_{\frac{\gamma_1}{2}}}\left(-k_z z_e - b_z \dot{z} + g\right)$$

$$u_2 = \frac{I_x}{a}\left(-k_\theta \theta_e - b_\theta \dot{\theta}\right)$$

$$u_3 = -2\theta + 2\tan^{-1}\left(\frac{-k_x x_e - b_x \dot{x}}{g}\right)$$

Simplified planar model



- Feedback linearization

We started with designing controllers for the simplified planar model by using Exact feedback linearization.

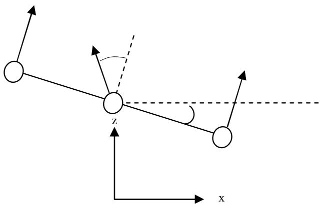
Bottom-up approach – Controllers

$$u_1 = \frac{M}{2C_{\theta+\frac{\gamma_1}{2}}C_{\frac{\gamma_1}{2}}}\left(-k_z z_e - b_z \dot{z} + g\right)$$

$$u_2 = \frac{I_x}{a}\left(-k_\theta \theta_e - b_\theta \dot{\theta}\right)$$

$$u_3 = -2\theta + 2\tan^{-1}\left(\frac{-k_x x_e - b_x \dot{x}}{g}\right)$$

Simplified planar model



$$u_1 = \frac{M}{2C_{\theta+\frac{\gamma_1}{2}}C_{\frac{\gamma_1}{2}}C_{\phi+\frac{\gamma_2}{2}}C_{\frac{\gamma_2}{2}}}\left(-k_z z_e - b_z \dot{z} + g\right)$$

$$u_2 = \frac{aI_x}{C_{\gamma_2}}\left(-k_\theta \theta_e - b_\theta \dot{\theta}\right)$$

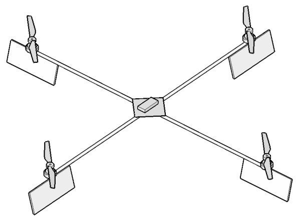
$$u_3 = \frac{aI_y}{C_{\gamma_1}}\left(-k_\phi \phi_e - b_\phi \dot{\phi}\right)$$

$$u_4 = -2\phi + 2\tan^{-1}\left(\frac{k_y y_e + b_y \dot{y}}{Mg}\right)$$

$$u_5 = -2\theta + 2\tan^{-1}\left(\frac{-k_x x_e - b_x \dot{x}}{Mg}\right)$$

$$u_6 = \sin^{-1}\left(-k_\psi \psi_e - b_\psi \dot{\psi}\right)$$

Simplified Model



- Feedback linearization

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Than we used the inspiration from the planar model to chose controllers for the full spatial model

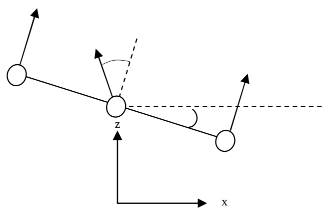
Bottom-up approach – Controllers

$$u_1 = \frac{M}{2C_{\theta+\frac{\gamma_1}{2}} C_{\frac{\gamma_1}{2}}}\left(-k_z z_e - b_z \dot{z} + g\right)$$

$$u_2 = \frac{I_x}{a}\left(-k_\theta \theta_e - b_\theta \dot{\theta}\right)$$

$$u_3 = -2\theta + 2\tan^{-1}\left(\frac{-k_x x_e - b_x \dot{x}}{g}\right)$$

Simplified planar model



$$u_1 = \frac{M}{2C_{\theta+\frac{\gamma_1}{2}} C_{\frac{\gamma_1}{2}} C_{\phi+\frac{\gamma_2}{2}} C_{\frac{\gamma_2}{2}}}\left(-k_z z_e - b_z \dot{z} + g\right)$$

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$$u_4 = -2\phi + 2\tan^{-1}\left(\frac{k_y y_e + b_y \dot{y}}{Mg}\right)$$

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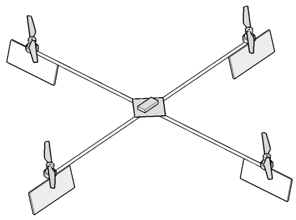
Simplified Model



The Full Model
(real world)

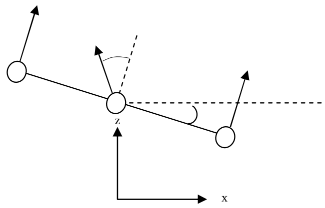
- Feedback linearization

this felt comfortable enough to implement on the real world system

Bottom-up approach – Closed Loop

$$\begin{Bmatrix} \ddot{x} \\ \ddot{z} \\ \ddot{\theta} \end{Bmatrix} = \begin{Bmatrix} \frac{1}{g}\left(-k_z z_e - b_z \dot{z} + g\right)\left(-k_x x_e - b_x \dot{x}\right) \\ -k_z z_e - b_z \dot{z} \\ -k_\theta \theta_e - b_\theta \dot{\theta} \end{Bmatrix}$$

Simplified planar model



- Asymptotically stable
- Locally stable

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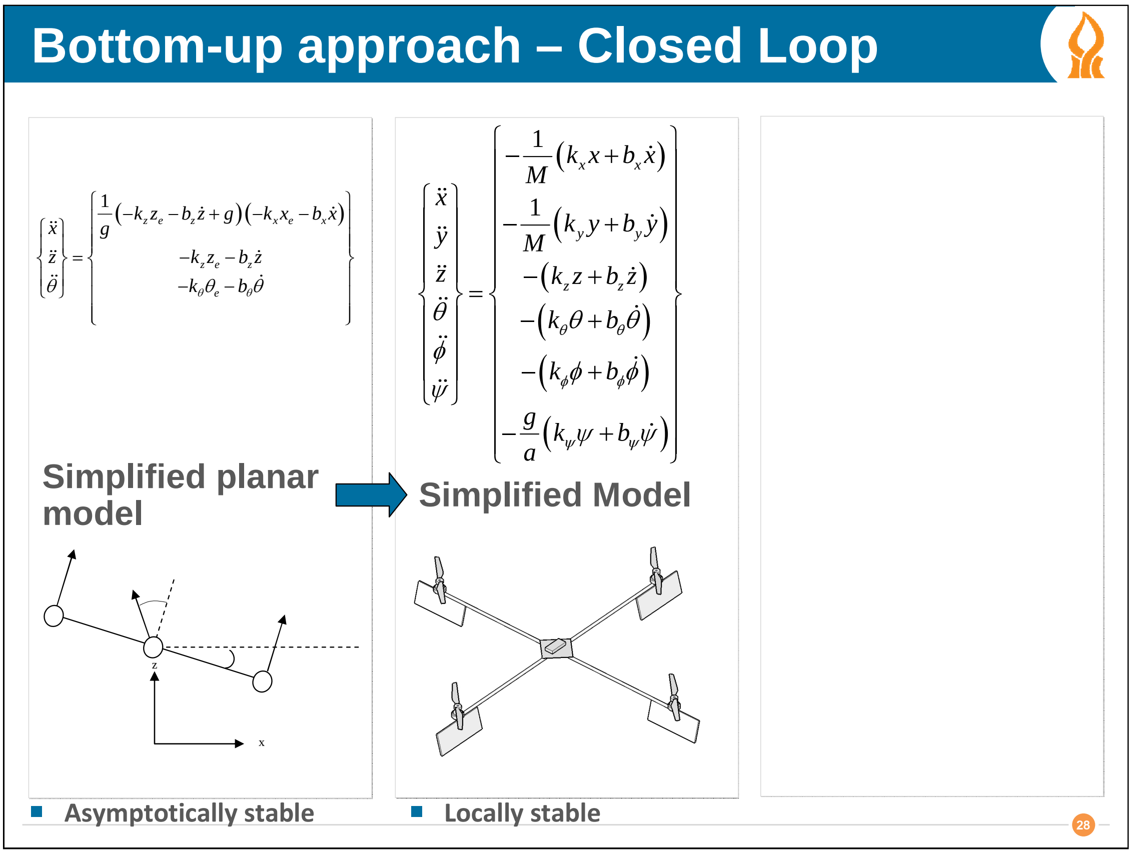
Of course we had to verify that the feeling is right so:

We substituted the controllers into the model to get a closed loop systems.

We went on and checked that the closed loop system of the planar model is asymptotically stable



and that for the full spatial model it is a least locally stable.

The expressions presented here for full closed loop system represent the linearized system description.

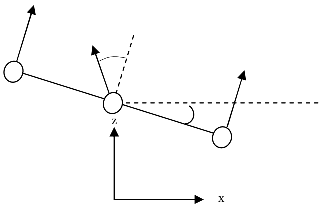
This is because the system non-linearized description is just too large to present.

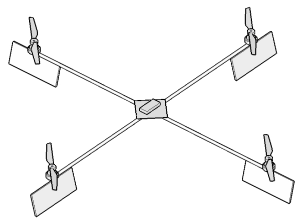
Bottom-up approach – Closed Loop

$$\begin{Bmatrix}\ddot{x}\\ \ddot{z}\\ \ddot{\theta}\end{Bmatrix} = \begin{Bmatrix}\frac{1}{g}\left(-k_z z_e - b_z\dot{z} + g\right)\left(-k_x x_e - b_x\dot{x}\right)\\ -k_z z_e - b_z\dot{z}\\ -k_\theta\theta_e - b_\theta\dot{\theta}\end{Bmatrix}$$

Simplified planar model → **Simplified Model** → **The Full Model** (real world)

$$\begin{Bmatrix}\ddot{x}\\ \ddot{y}\\ \ddot{z}\\ \ddot{\theta}\\ \ddot{\phi}\\ \ddot{\psi}\end{Bmatrix} = \begin{Bmatrix}-\frac{1}{M}\left(k_x x + b_x\dot{x}\right)\\ -\frac{1}{M}\left(k_y y + b_y\dot{y}\right)\\ -\left(k_z z + b_z\dot{z}\right)\\ -\left(k_\theta\theta + b_\theta\dot{\theta}\right)\\ -\left(k_\phi\phi + b_\phi\dot{\phi}\right)\\ -\frac{g}{a}\left(k_\psi\psi + b_\psi\dot{\psi}\right)\end{Bmatrix}$$





- Asymptotically stable
- Locally stable

That stage was a good time to go and check this further by numerical simulations.

Simulations

Simulations

- Validating the closed loop performance of the control system with the **non-linear dynamic model** of the FAR
- Verify the closed loop **system stability**.
- Introduce **other non-linear properties** to the model that were not included in the system's mathematical analysis.
  - Saturation of the thrust and saturation of the thrust deflection angle
- Variety of initial conditions.

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We used simulations to validate that the control system we designed

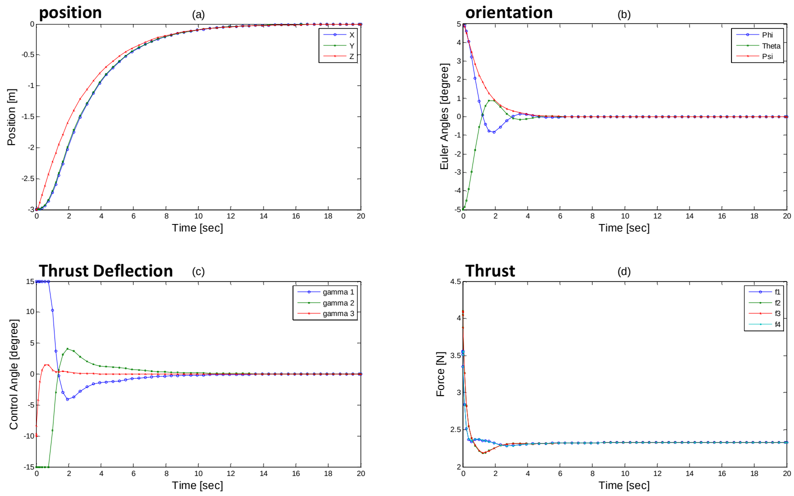
still worked well for the non reduced. non-linear model.

We also used the simulations to add non-linear properties that were not included in the analytical model

Such as the saturation of actuators

Non zero I.C. on all DOFs

- **I.C.** $X = -3; Y = -3; Z = -3; \phi = 5^0; \theta = -5^0; \psi = 5$
- **The combined (position and orientation) I.C. causes actuator saturation**



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In the following example you can see how from

Non zero initial conditions on all DOFs

The FAR converge smoothly to equilibrium both in orientation and in position

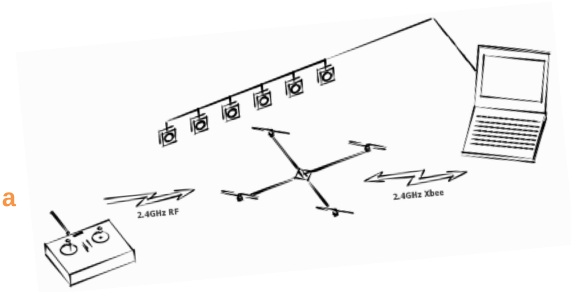
Even though you can notice the saturation of the thrust deflection actuator

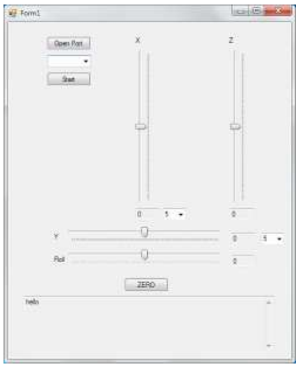
Experiments

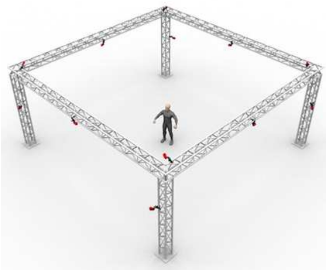
After we tested our system with simulations we were ready to build an experimental system

Experimental system

- System parameters identification
- Flight tests (perturbations)
- Compare FAR and quad
- Optitrack camera system serves as a positioning system (10Hz) and data logger (100Hz)
- Avionics and flight control (200Hz) – Open Source ArdupilotMega + IMU Shield
- Test arena 4 x 3 x 3m







We build an experimental system and used it to:

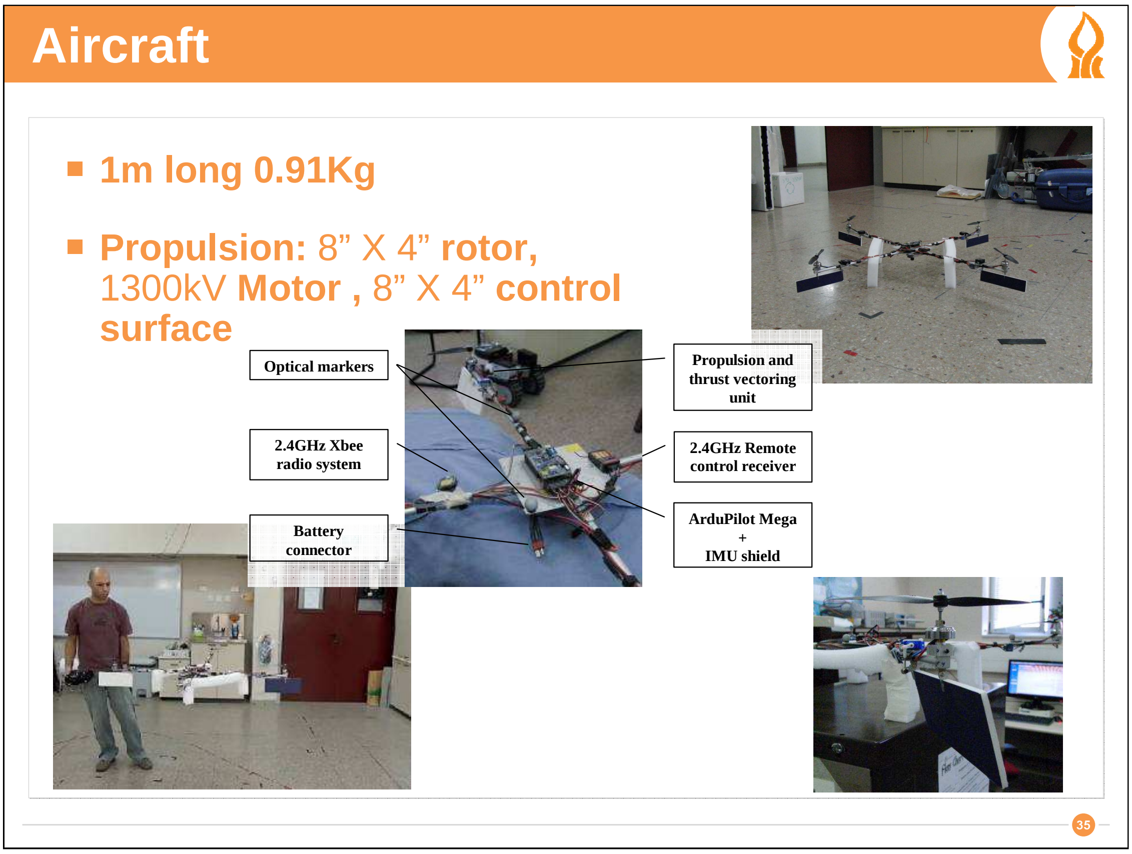
Identify system parameters

conduct flight tests

And to compare the performances of FAR and QUAD

The experimental system test arena is 4 X 3 X 3 meters in size (no so big)
it is surrounded by 12 cameras that are connected to a PC.
The PC also ran a control consol.

Naturally the system also included an aircraft and RC radio as a safety feature

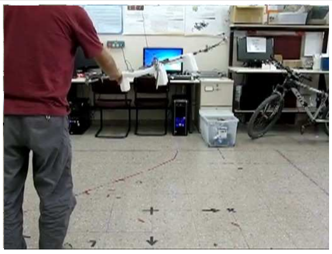


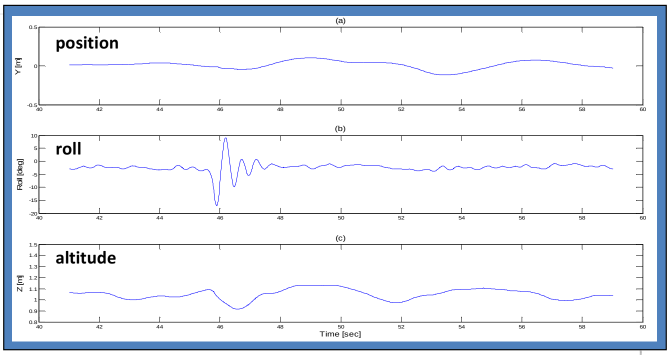
The FAR aircraft we used is 1 meter long

Which is long considering the size of rotors we used

And it weighs approximately 900 grams

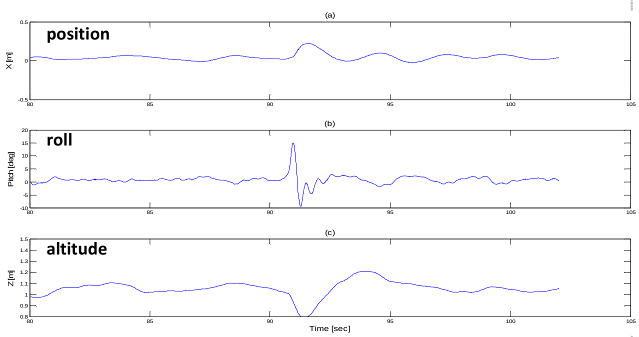
Flight tests – roll perturbation





FAR – the aerodynamic actuator range is larger than assumed in simulation

QUAD using a back stepping controller $\theta = k_\theta x + b_\theta \dot{x}$

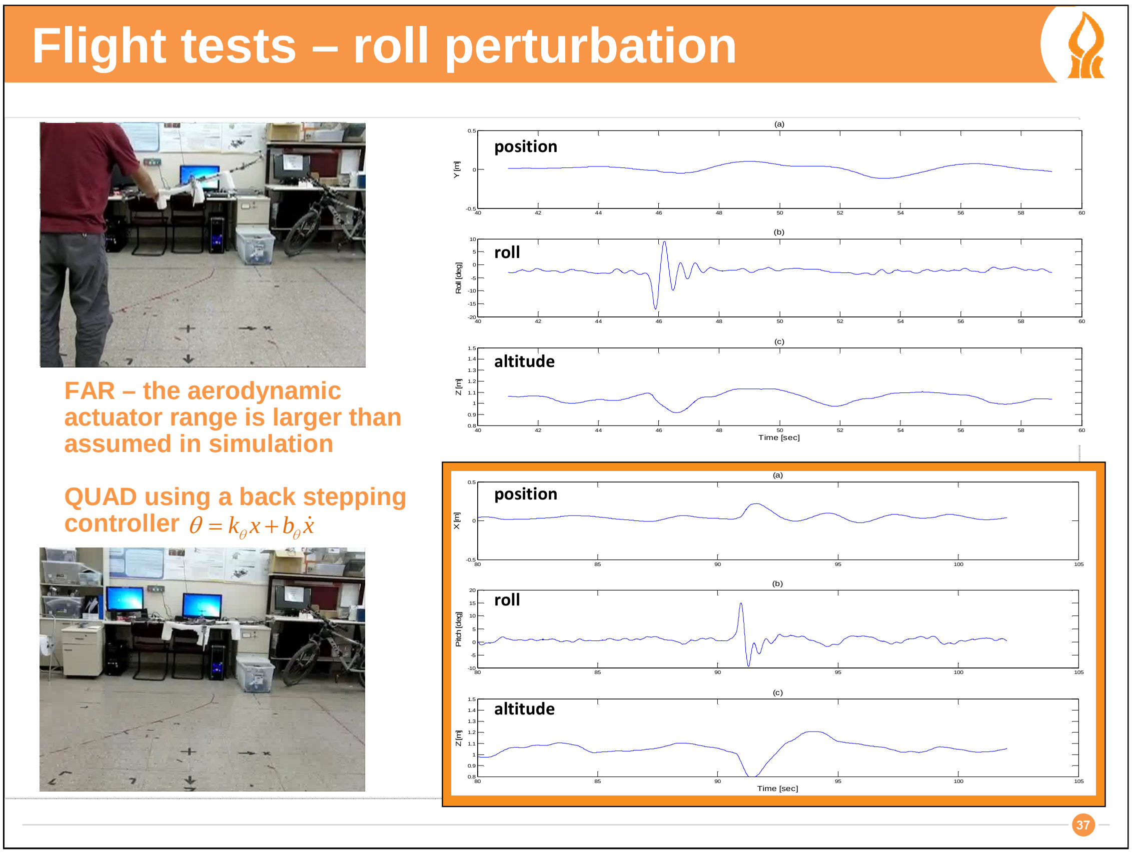


Now for the real thing – Flight Tests!

In the video running in the top left you can see me perturbing the FAR.

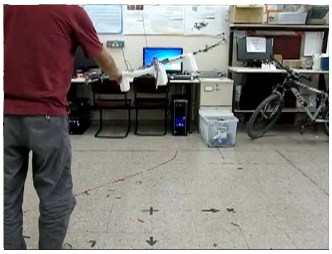
You can see how the control surfaces work to compensate for the roll and how the FAR hardly changes position or altitude.

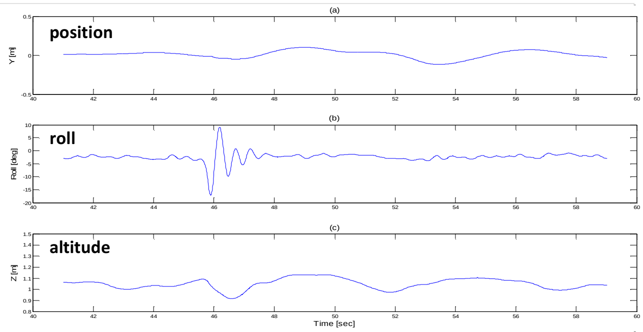
You can also see this very clearly on the graphs



On the bottom half of the slide you can see what happens when the same thing is done to a QUADROTOR

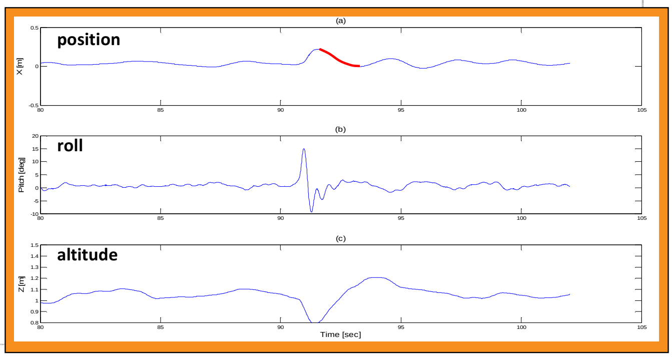
Flight tests – roll perturbation



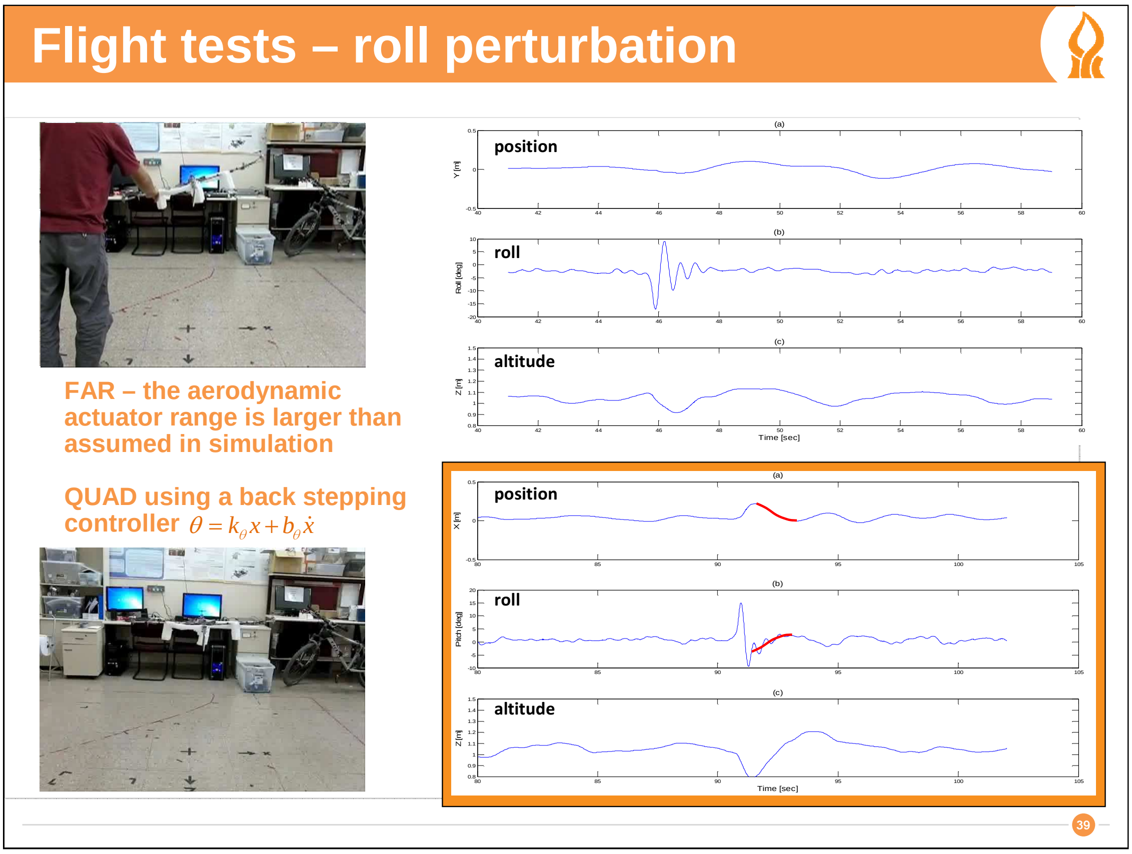


FAR – the aerodynamic actuator range is larger than assumed in simulation

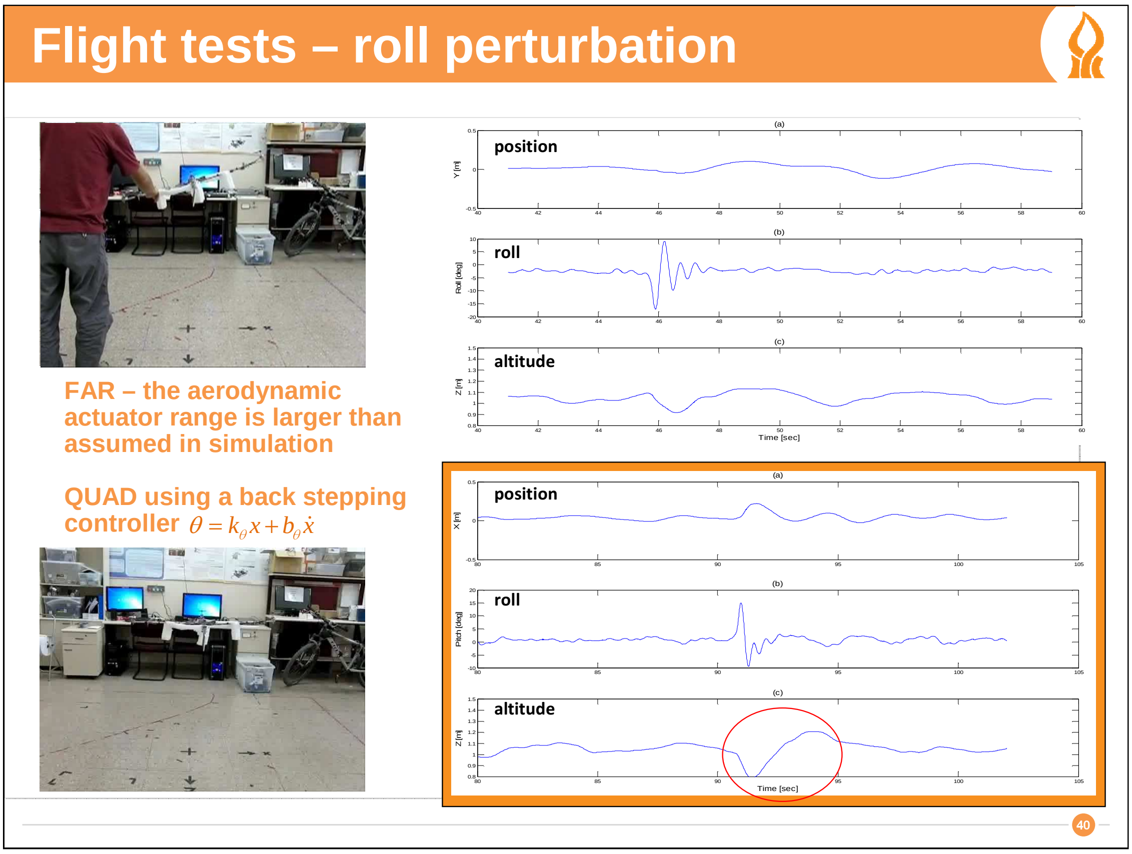
QUAD using a back stepping controller $\theta = k_\theta x + b_\theta \dot{x}$



1st – it moves considerably

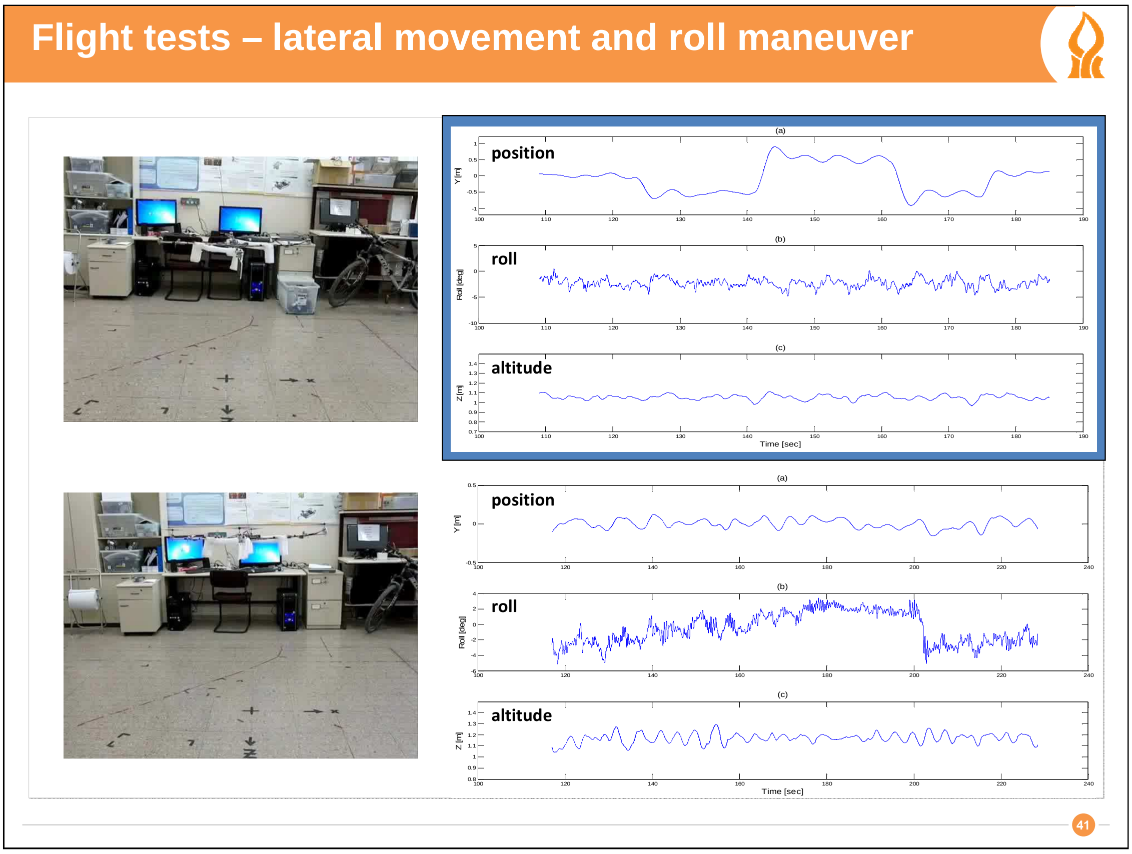


2nd – I would like to draw your attention to the graphs where you can see how it moved and that because of the back stepping position controller the roll angle converges to zero along a path set by the lateral position.

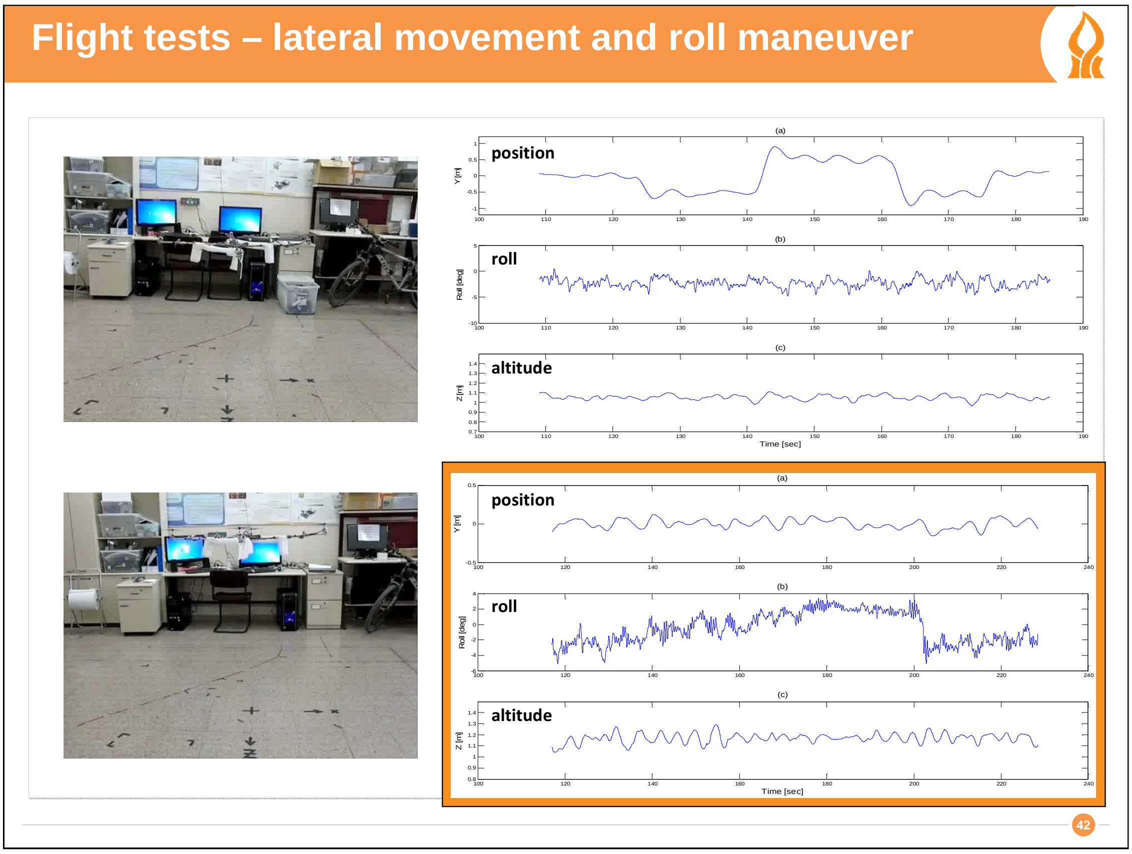


3rd – It also loses altitude; although this can be compensated it will come at the cost of a larger lateral movement.

Ok Let's move on!



In the top half of this slide you can see a repeated lateral movement.
You can see that the FAR does not roll or change altitude during this maneuver
Again you can see how the control surfaces are employed to achieve that.



In the other half of the slide

You can see how it rolls without changing its position

Please take a few seconds to watch it

Conclusion

- FAR - structure, model and control
- Advantages:
  - disturbance rejection
  - larger yaw moment allow flexibility in the design
  - Larger configuration space
- Future work:
  - Improved control system
  - aero-mechanical optimal design

For conclusion:

I have presented:

The FAR

Its structure, analytical model and the control system we have designed for it.

It has advantages in:

Disturbance rejection

and

The larger yaw torque allow for grater flexibility in the design.

For future work we intend to:

Improve the control system

and

Optimize the aero-mechanical design

Thank You!